THEORY OF LIMIT EQUILIBRIUM OF BIMETALLIC SHELLS OF REVOLUTION AND CIRCULAR PLATES

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Bimetallic shells and plates are widely used in technology (see [1, 2]). An investigation into the flexure and stability of thin shells and various types of loading within the limits of elasticity has been carried out in [3]. An investigation into the load-carrying capacity of cylindrical bimetallic shells made of materials which equally resist tension and compression was carried out in [4]. In many cases the materials of the base and plating layers of bimetallic constructions possess substantially different plastic resistance under tension and compression [5]. The given paper is devoted to the investigation of the load-carrying capacity of bimetallic axisymmetric shells which are made of materials that have different resistances to tension and compression; it is also devoted to the assessment of their economy in comparison with homogeneous shells.

1. We consider thin bimetallic shells of revolution, the two layers of which are made of different ideally rigid-plastic materials with nonidentical yield points under tension and compression.

The surface separating the layers of the shell is taken as the reference surface, and we direct the Z axis along the inner normal to this surface.

We introduce the notation

$$s_i' = \frac{\sigma_i'}{\sigma_0}, \quad s_i'' = \frac{\sigma_i''}{\sigma_0}, \quad \gamma^{\pm} = \frac{\sigma_s^{\pm}}{\sigma_0}, \quad s^{\pm} = \frac{k_s^{\pm}}{\sigma_0}$$

where σ_i', σ_i'' ($i=1, 2$) are the principal stresses acting in the upper and lower layers; $\sigma_s^{\pm}$, $k_s^{\pm}$ are the yield points of the materials under tension (plus) and compression (minus) of the upper and lower layers respectively; σ_0 is the yield point of a certain conventional material.

We assume that the materials of the layers are rigidly joined to one another and each of them in the limit state satisfies the plasticity condition of P. P. Balandin [6] which is linearized according to the type of linearization of the Mises ellipse by the Tresca hexagon (Fig. 1). Here we have introduced the notation

$$\gamma_* = (\gamma^{+2} - \gamma^+\gamma^- + \gamma^{-2})^{1/2}, \quad \Delta_1 = \gamma^- - \gamma^+$$
$$s_* = (s^{+2} - s^+s^- + s^{-2})^{1/2}, \quad \Delta_2 = s^- - s^+$$

Assuming that the Kirchhoff–Love hypothesis is valid for the entire cross section of the shell, we express the principal strain rates $\dot{\varepsilon}_i$ ($i=1, 2$) in terms of the strain rates $\dot{\varepsilon}_{i0}$ and the curvature rates $\dot{\varkappa}_i$ of points of the reference surface

$$\dot{\varepsilon}_i = \dot{\varepsilon}_{i0} - z\dot{\gamma}_i, \quad z = 2Z/(h_1 + h_2), \quad \dot{\gamma}_i = {}^1/_2(h_1 + h_2)\dot{\varkappa}_i$$

where h_1 and h_2 and the thicknesses of the upper and lower layers of the shell.

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The intensities of the forces and moments acting in the section of the shell are introduced by the expressions

$$T_i = \int_{-h_1}^{0} \sigma_i' dZ + \int_{0}^{h_2} \sigma_i'' dZ, \quad M_i = \int_{-h_1}^{0} \sigma_i' Z dZ + \int_{0}^{h_2} \sigma_i'' Z dZ, \quad i = 1, 2 \tag{1.1}$$

Assuming that the shell takes up all the load-carrying capacity when both its constituent layers reach the limit state, using the plasticity conditions and the associated flow rule adopted, it is not difficult to establish, just as in [7], the stress distribution in the limit state of the shell. The substitution of these stresses into (1.1), after certain transformation enables us to obtain the final relationships between the forces and moments in the limit state of the section of the shell.

In the following it is convenient to use the following dimensionless quantities

$$t_i = \frac{T_i}{\sigma_0 h_1 (1+\alpha)} = \frac{2}{1+\alpha}\left(t_i^* - \frac{\Delta_1 + \alpha\Delta_2}{2}\right),$$
$$m_i = \frac{4M_i}{\sigma_0 h_1^2 (1+\alpha)^2} = \left(m_i^* - \frac{\alpha^2\Delta_2 - \Delta_1}{2}\right)\frac{4}{(1+\alpha)^2}$$
$$t_i^* \pm {}^1/_2 (\alpha s_* - \gamma_*) = t_i^\pm, \quad m_i^* \pm {}^1/_2 (\alpha^2 s_* + \gamma_*) = m_i^\pm, \quad \alpha = h_2 / h_1$$
$$\beta_i^\pm = 2\,[\pm m_i^\pm s_* - (t_i^\pm)^2]^{1/2}, \quad \delta_i^\pm = 2\,[\pm m_i^\pm \gamma_* - (t_i^\pm)^2]^{1/2}, \quad i = 1, 2$$
$$\beta_{ij}^\pm = 2\,[\pm (m_i^\pm - m_j^*)\, s_* - (t_i^\pm - t_j^*)^2]^{1/2},$$
$$\delta_{ij}^\pm = 2\,[\pm (m_i^\pm - m_j^*)\, \gamma_* - (t_i^\pm - t_j^*)^2]^{1/2}, \quad i, j = 1, 2;\ i \neq j$$

Plastic states corresponding to the sides of the hexagons in Fig. 1 will be called regular states, while states corresponding to the corners will be called singular states. Then the final relationships and the corresponding flow rules, for cases where regular, or again the maximum number of singular plastic states, are realized across the thickness of the shell (in the brackets we have shown the plastic states corresponding to Fig. 1a, b and measured from the upper edge of the shell, as well as the parameters z_{01}, z_{02}, z_{01}', z_{02}', l, m, n, p, q, r, which separate these states), have the form

$$\begin{gathered}(A_1 B_1 z_{02} D_1 E_1,\ D_2 E_2) \text{ or } (D_1 E_1 z_{02} A_1 B_1 A_2 B_2) \text{ for } i = 2\\ (A_1 F_1 z_{01} C_1 D_1,\ C_2 D_2) \text{ or } (C_1 D_1 z_{01} A_1 F_1,\ A_2 F_2) \text{ for } i = 1\\ m_i^\pm = \pm\frac{(t_i^\pm)^2}{\gamma_*}; \quad z_{0i} = \pm\frac{t_i^\pm}{\gamma_*}; \quad z_{02} = q = r, \quad p = \frac{0}{0};\\ z_{01} = p = q, \quad r = \frac{0}{0}\end{gathered} \tag{1.2}$$

$$\begin{gathered}(A_1 B_1,\ A_2 B_2 z_{02}' D_2 E_2) \text{ or } (D_1 E_1,\ D_2 E_2 z_{02}' A_2 B_2) \text{ for } i = 2\\ (A_1 F_1,\ A_2 F_2 z_{01}' C_2 D_2) \text{ or } (C_1 D_1,\ C_2 D_2 z_{01}' A_2 F_2) \text{ for } i = 1\\ m_i^\pm = \pm\frac{(t_i^\pm)^2}{s_*}; \quad z_{0i}' = \pm\frac{t_i^\pm}{s_*}; \quad z_{02}' = m = n,\\ l = \frac{0}{0}; \quad z_{01}' = l = m, \quad n = \frac{0}{0}\end{gathered} \tag{1.3}$$

$$\begin{gathered}(B_1 C_1 z_{03} E_1 F_1,\ E_2 F_2) \text{ or } (E_1 F_1 z_{03} B_1 C_1,\ B_2 C_2) \text{ for } \delta = \gamma_*\\ (B_1 C_1,\ B_2 C_2 z_{03}' E_2 F_2) \text{ or } (E_1 F_1,\ E_2 F_2 z_{03}' B_2 C_2) \text{ for } \delta = s_*\\ m_2^\pm - m_1^* = \pm\frac{(t_2^\pm - t_1^*)^2}{\delta}, \quad z_{03} = p = r = \pm\frac{t_2^\pm - t_1^*}{\gamma_*},\\ q = \frac{0}{0}, \quad z_{03}' = l = n = \pm\frac{t_2^\pm - t_1^*}{s_*}, \quad m = \frac{0}{0}\end{gathered} \tag{1.4}$$

$(A_1 p B_1 q C_1 r D_1 D_2)$, $(A_1 r F_1 q E_1 p D_1 D_2)$ or $(D_1 r C_1 q B_1 p A_1,\ A_2)$
$(D_1 p E_1 q F_1 r A_1,\ A_2)$ for $\delta = \gamma_*$; $(A_1 A_2 l B_2 m C_2 n D_2)$,
$(A_1 A_2 n F_2 m E_2 l D_2)$ or $(D_1, D_2 n C_2 m B_2 l A_2)$, $(D_1,\ D_2 l E_2 m F_2 n A_2)$

for $\delta = s_*$

$$m_2^\pm = \pm\frac{\delta}{4}\left[\left(\frac{m_1^\pm - m_2^\pm}{t_1^\pm - t_2^\pm}\right)^2 \mp \frac{4t_1^\pm}{\delta}\,\frac{m_1^\pm - m_2^\pm}{t_1^\pm - t_2^\pm} + \frac{4}{\delta^2}\left(t_1^{\pm 2} + t_2^{\pm 2}\right)\right] \tag{1.5}$$

$$p = \frac{1}{2}\left[\frac{m_1^{\pm} - m_2^{\pm}}{t_1^{\pm} - t_2^{\pm}} \pm \frac{2}{\gamma_*}(t_1^{\pm} - t_2^{\pm})\right],$$
$$q = -\frac{1}{2}\left[\frac{m_1^{\pm} - m_2^{\pm}}{t_1^{\pm} - t_2^{\pm}} \mp \frac{2}{\gamma_*}(t_1^{\pm} + t_2^{\pm})\right]$$
$$r = \frac{1}{2}\left[\frac{m_1^{\pm} - m_2^{\pm}}{t_1^{\pm} - t_2^{\pm}} \mp \frac{2}{\gamma_*}(t_1^{\pm} - t_2^{\pm})\right]$$

$(C_1rD_1pE_1qF_1,\ F_2)$, $(C_1qB_1pA_1rF_1F_2\ )$ or $(F_1qE_1pD_1rC_1C_2)$ $(F_1rA_1pB_1qC_1,\ C_2)$ for $\delta = \gamma_*$; $(C_1,\ C_2nD_2lE_2mF_2)$, $(C_1C_2mB_2lA_2nF_2)$ for $(F_1,F_2mE_2lD_2nC_2)$, $(F_1,\ F_2nA_2lB_2mC_2)$ for $\delta = s_*$

$$m_1^{\mp} = \pm\frac{\delta}{2}\left[\frac{1}{2}\left(\frac{m_2^*}{t_2^*} \mp \frac{2t_2^*}{\delta}\right)^2 \pm \frac{2t_1^{\mp}}{\delta}\left(\frac{m_2^*}{t_2^*} \mp \frac{2t_2^*}{\delta}\right) + \frac{4}{\delta^2}t_1^{\mp 2}\right]$$
$$p = -\frac{1}{2}\left(\frac{m_2^*}{t_2^*} \mp \frac{2t_2^*}{\gamma_*}\right) \mp \frac{2t_1^{\mp}}{\gamma_*}, \quad q = \frac{1}{2}\left(\frac{m_2^*}{t_2^*} \mp \frac{2t_2^*}{\gamma_*}\right),$$
$$r = \frac{1}{2}\left(\frac{m_2^*}{t_2^*} \pm \frac{2t_2^*}{\gamma_*}\right) \tag{1.6}$$

$(B_1,\ B_2mC_2nD_2lE_2)$, $(B_1,B_2lA_2nF_2mE_2)$ or $(E_1,\ E_2lD_2nC_2mB_2)$ $(E_1,\ E_2mF_2nA_2lB_2)$ for $\delta = s_*$; $(B_1qC_1rD_1pE_1,\ E_2)$, $(B_1pA_1rF_1qE_1,\ E_2)$ or $(E_1pD_1rC_1qB_1,\ B_2)$, $(E_1qF_1rA_1pB_1,\ B_2)$ for $\delta = \gamma_*$

$$m_2^{\pm} = \pm\frac{\delta}{2}\left[\frac{1}{2}\left(\frac{m_1^*}{t_1^*} \pm \frac{2t_1^*}{\delta}\right)^2 \mp \frac{2t_2^{\pm}}{\delta}\left(\frac{m_1^*}{t_1^*} \pm \frac{2t_1^*}{\delta}\right) + \frac{4}{\delta^2}t_2^{\pm 2}\right]$$
$$p = \frac{1}{2}\left(\frac{m_1^*}{t_1^*} \mp \frac{2t_1^*}{\gamma_*}\right), \quad q = \frac{1}{2}\left(\frac{m_1^*}{t_1^*} \pm \frac{2t_1^*}{\gamma_*}\right), \tag{1.7}$$
$$r = -\frac{1}{2}\left(\frac{m_1^*}{t_1^*} \pm \frac{2t_1^*}{\gamma_*}\right) \pm \frac{2t_2^{\pm}}{\gamma_*}$$

The expressions for the parameters l, m, n in the relations (1.5)-(1.7) are obtained from the relationships for the parameters p, q, r by replacing in them γ_* by s_* (A_1, B_1, B_2, C_2, D_2) or (D_1, E_1, E_2, F_2, A_2), $[-2/(1+\alpha) < p < 0 < m < n < 2\alpha/(1+\alpha)]$ for $i=1$, $j=2$, $\eta_1 = p$, $\eta_2 = m$, $\eta_3 = n$; (A_1, F_1, F_2, E_2, D_2) or (D_1, C_1, C_2, B_2, A_2) $[-2/(1+\alpha) < r < 0 < m < l < 2\alpha/(1+\alpha)]$ for $i=2$, $j=1$, $\eta_1 = r$, $\eta_2 = m$, $\eta_3 = l$

$$m_i^{\pm} = \pm\frac{1}{8}\left\{\frac{1}{\gamma_*}[\pm 2(2t_i^{\pm} - t_j^{\pm}) + \beta_j^{\pm}]^2 + \frac{1}{s_*}(\pm 2t_j^{\pm} - \beta_j^{\pm})^2\right\} \tag{1.8}$$
$$\eta_1 = \frac{1}{2\gamma_*}[\pm 2(2t_i^{\pm} - t_j^{\pm}) + \beta_j^{\pm}], \quad \eta_2 = \frac{1}{2s_*}(\pm 2t_j^{\pm} - \beta_j^{\pm}),$$
$$\eta_3 = \frac{1}{2s_*}(\pm 2t_j^{\pm} + \beta_j^{\pm})$$

$(E_1,\ D_1,\ D_2,\ C_2,\ B_2)$ or $(B_1,\ A_1,\ A_2,\ F_2,\ E_2)$, $(-2/(1+\alpha) < p < 0 < n < m < 2\alpha/(1+\alpha))$ for $i = 1$, $j = 2$, $\eta_1 = p$, $\eta_2 = m$, $\eta_3 = n$; $(C_1,\ D_1,\ D_2,\ E_2,\ F_2)$ or $(F_1,\ A_1,\ A_2,\ B_2,\ C_2)$, $(-2/(1+\alpha) < r < 0 < l < m < 2\alpha/(1+\alpha))$ for $i = 2$, $j = 1$, $\eta_1 = r$, $\eta_2 = m$, $\eta_3 = l$

$$m_i^* = \pm\frac{1}{8}\left\{\frac{1}{\gamma_*}[\pm 2(2t_i^* - t_j^{\mp}) + \beta_j^{\mp}]^2 - \frac{1}{s_*}(\mp 2t_j^{\mp} + \beta_j^{\mp})^2\right\} \tag{1.9}$$
$$\eta_1 = \frac{1}{2\gamma_*}[\pm 2(2t_i^* - t_j^{\mp}) + \beta_j^{\mp}], \quad \eta_2 = \frac{1}{2s_*}(\mp 2t_j^{\mp} + \beta_j^{\mp}),$$
$$\eta_3 = \frac{1}{2s_*}(\mp 2t_j^{\mp} - \beta_j^{\mp})$$

$(B_1,\ C_1,\ C_2,\ D_2,\ E_2)$ or $(E_1,\ F_1,\ F_2,\ A_2,\ B_2)$, $(-2/(1+\alpha) < q < 0 < n < l < 2\alpha/(1+\alpha))$ for $i = 1$, $j = 2$, $\eta_1 = q$, $\eta_2 = l$, $\eta_3 = n$; $(F_1,\ E_1,\ E_2,\ D_2,\ C_2)$ for $(C_1,\ B_1,\ B_2,\ A_2,\ F_2)$ $(-2/(1+\alpha) < q < 0 < l < n < 2\alpha/(1+\alpha))$ for $i = 2$, $j = 1$, $\eta_1 = q$, $\eta_2 = n$, $\eta_3 = l$

$$m_i^* = \pm{}^1/_8\{\gamma_*^{-1}[\pm 2(t_i^* + t_j^{\pm}) + \beta_{ji}^{\pm}]^2 - s_*^{-1}[\pm 2(t_j^{\pm} - t_i^*) + \beta_{ji}^{\pm}]^2\}$$
$$\eta_1 = {}^1/_2\gamma_*^{-1}[\pm 2(t_i^* + t_j^{\pm}) + \beta_{ji}^{\pm}], \quad \eta_2 = {}^1/_2 s_*^{-1}[\pm 2(t_j^{\pm} - t_i^*) + \beta_{ji}^{\pm}], \tag{1.10}$$
$$\eta_3 = {}^1/_2 s_*^{-1}[\pm 2(t_j^{\pm} - t_i^*) - \beta_{ji}^{\pm}]$$

$(C_1,\ D_1,\ E_1,\ E_2,\ F_2)$ or $(F_1,\ A_1,\ B_1,\ B_2,\ C_2)$, $(-2/(1+\alpha) < r < p < 0 < m + 2\alpha/(1+\alpha))$ for $i = 2$, $j = 1$, $\eta_1 = p$, $\eta_2 = r$, $\eta_3 = m$; $(E_1,\ D_1,\ C_1,\ C_2,\ B_2)$ for $(B_1\ A_1,\ F_1,\ F_2,\ E_2)$ $(-2/(1+\alpha) < p < r < 0 < m < 2\alpha/(1+\alpha))$ for $i = 1$, $j = 2$, $\eta_1 = r$, $\eta_2 = p$, $\eta_3 = m$

$$m_i^* = \pm {}^1/_8 \{\gamma_*^{-1} [\pm 2 (t_i^* - t_j^\mp) - \delta_{ji}^\mp]^2 - s_*^{-1} [\mp 2 (t_i^* + t_j^\mp) - \delta_{ji}^\mp]^2\}$$

$$\eta_1 = {}^1/_2 \gamma_*^{-1} [\pm 2 (t_i^* - t_j^\mp) + \delta_{ji}^\mp], \quad \eta_2 = {}^1/_2 \gamma_*^{-1} [\pm 2 (t_i^* - t_j^\mp) - \delta_{ji}^\mp],$$

$$\eta_3 = {}^1/_2 s_*^{-1} [\mp 2 (t_i^* + t_j^\mp) - \delta_{ji}^\mp] \tag{1.11}$$

$(A_1, B_1, C_1, C_2, D_2)$ or $(D_1, E_1, F_1, F_2, A_2)$, $(-2/(1+\alpha) < p < q < 0 < n < 2\alpha/(1+\alpha))$ for $i = 2$, $j = 1$, $\eta_1 = p$, $\eta_2 = q$, $\eta_3 = n$; $(A_1, F_1, E_1, E_2, D_2)$ or $(D_1, C_1, B_1, B_2, A_2)$ $(-2/(1+\alpha) < r < q < 0 < l < 2\alpha/(1+\alpha))$ for $i = 1$, $j = 2$, $\eta_1 = r$, $\eta_2 = q$, $\eta_3 = l$

$$m_i^\pm = {}^1/_8 \{\gamma_*^{-1} [\pm 2t_j^\pm + \delta_j^\pm]^2 + s_*^{-1} [\pm 2 (2t_i^\pm - t_j^\pm) - \delta_j^\pm]^2\}$$

$$\eta_1 = {}^1/_2 \gamma_*^{-1} (\pm 2t_j^\pm - \delta_j^\pm), \quad \eta_2 = {}^1/_2 \gamma_*^{-1} (\pm 2t_j^\pm + \delta_j^\pm),$$

$$\eta_3 = {}^1/_2 s_*^{-1} [\pm 2 (2t_i^\pm - t_j^\pm) - \delta_j^\pm] \tag{1.12}$$

$(F_1, E_1, D_1, D_2, C_2)$ or $(C_1, B_1, A_1, A_2, F_2)$ $(-2/(1+\alpha) < q < p < 0 < n < 2\alpha/(1+\alpha))$ for $i = 2$, $j = 1$, $\eta_1 = p$, $\eta_2 = q$, $\eta_3 = n$; $(B_1, C_1, D_1, D_2, E_2)$ or $(E_1, F_1, A_1, A_2, B_2)$ $(-2/(1+\alpha) < q < r < 0 < l < 2\alpha/(1+\alpha))$ or $i = 1$, $j = 2$, $\eta_1 = r$, $\eta_2 = q$, $\eta_3 = l$

$$m_i^* = \pm {}^1/_8 \{\gamma_*^{-1} (\pm 2t_j^\pm - \delta_j^\pm)^2 - s_*^{-1} [\mp 2 (2t_i^* - t_j^\pm) - \delta_j^\pm]^2\}$$

$$\eta_1 = {}^1/_2 \gamma_*^{-1} (\pm 2t_j^\pm + \delta_j^\pm), \quad \eta_2 = {}^1/_2 \gamma_*^{-1} (\pm 2t_j^\pm - \delta_j^\pm), \tag{1.13}$$

$$\eta_3 = {}^1/_2 s_*^{-1} [\mp 2 (2t_i^* - t_j^\pm) - \delta_j^\pm]$$

In the expressions presented above the upper sign corresponds to the distribution of the plastic states indicated in the first place, while the lower sign corresponds to the distribution of the states indicated in the second place. The parameters p, q, r, l, m, n, determining the ordinates of the surfaces which separate the different plastic states across the thickness of the shell, are introduced as follows:

$$p = \frac{\dot{\varepsilon}_{10}}{\dot{\gamma}_1}, \quad q = \frac{\dot{\varepsilon}_{10} + \dot{\varepsilon}_{20}}{\dot{\gamma}_1 + \dot{\gamma}_2}, \quad r = \frac{\dot{\varepsilon}_{20}}{\dot{\gamma}_2}, \quad -\frac{2}{1+\alpha} \leqslant z \leqslant 0$$

$$l = \frac{\dot{\varepsilon}_{10}}{\dot{\gamma}_1}, \quad m = \frac{\dot{\varepsilon}_{10} + \dot{\varepsilon}_{20}}{\dot{\gamma}_1 + \dot{\gamma}_2}, \quad n = \frac{\dot{\varepsilon}_{20}}{\dot{\gamma}_2}, \quad 0 \leqslant z \leqslant \frac{2\alpha}{1+\alpha}$$

In each of the cases (1.2)-(1.13) these parameters must satisfy certain inequalities giving rise to constraints for the corresponding portion of the limit hypersurface of yield.

2. When solving certain particular cases we can use three-dimensional yield surfaces, which considerably simplifies the process of selecting plastic states. In Fig. 2 we have shown the yield surface for a symmetrically loaded cylindrical shell ($\dot{\gamma}_2 = 0$). We shall present the equations constraining the parts of its surface.

The parts obtained by means of regular states of the yield conditions in terms of stresses (Fig. 1):

$$m_1^* = \frac{\alpha^2 s_* + \gamma_*}{2} - \frac{1}{4s_*} (-2t_1^* + \alpha s_* - \gamma_*)^2,$$

$$-\frac{\gamma_* + \alpha s_*}{2} \leqslant t_1^* \leqslant \frac{\alpha s_* - \gamma_*}{2} \tag{2.1}$$

$$m_1^* = -\frac{\alpha^2 s_* + \gamma_*}{2} + \frac{1}{4\gamma_*} (2t_1^* + \alpha s_* - \gamma_*)^2,$$

$$-\frac{\gamma_* + \alpha s_*}{2} \leqslant t_1^* \leqslant \frac{\gamma_* - \alpha s_*}{2} \tag{2.2}$$

$$m_1^* = -\frac{\alpha^2 s_* + \gamma_*}{2} + \frac{1}{4s_*} (2t_1^* + \alpha s_* - \gamma_*)^2, \quad \frac{\gamma_* - \alpha s_*}{2} \leqslant t_1^* \leqslant \frac{\gamma_* + \alpha s_*}{2} \tag{2.3}$$

$$m_1^* = \frac{\alpha^2 s_* + \gamma_*}{2} - \frac{1}{4\gamma_*} (-2t_1^* + \alpha s_* - \gamma_*)^2, \quad \frac{\alpha s_* - \gamma_*}{2} \leqslant t_1^* \leqslant \frac{\gamma_* + \alpha s_*}{2} \tag{2.4}$$

The parts obtained by means of singular plastic states:

$$t_2^* = (\alpha s_* + \gamma_*)/2, \; 0 \leqslant t_1^* \leqslant (\alpha s_* + \gamma_*)/2 \tag{2.5}$$

$$t_2^* - t_1^* = (\alpha s_* + \gamma_*)/2, \; -(\alpha s_* + \gamma_*)/2 \leqslant t_1^* \leqslant 0 \tag{2.6}$$

$$m_1^* = (\alpha^2 s_* + \gamma_*)/2 - {}^1/_2 s_*^{-1} \{[2(t_2^* - t_1^*) - \gamma_*]^2 + (-2t_2^* + \alpha s_*)^2\} \tag{2.7}$$

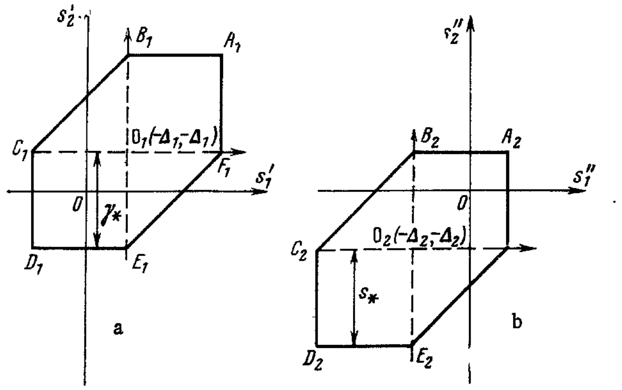

Fig. 1

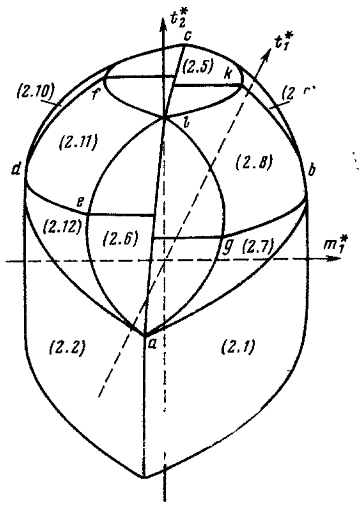

Fig. 2

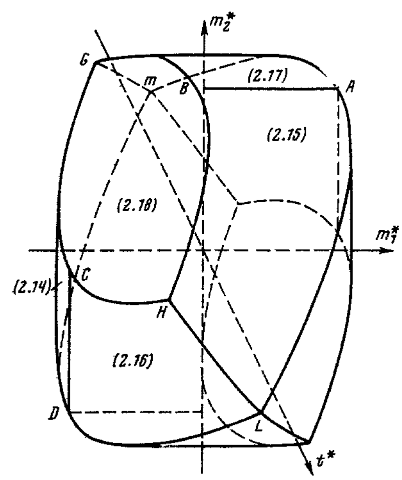

Fig. 3

$$m_1^* = (\alpha^2 s_* + \gamma_*)/2 - {}^1/_2\,\{\gamma_*^{-1}(-2t_2^* + \alpha s_*)^2{}_* + s^{-1}[2(t_2^* - t_1^*) - \gamma_*]^2\} \tag{2.8}$$

$$m_1^* = (\alpha^2 s_* + \gamma_*)/2 - {}^1/_2\gamma_*^{-1}\{[2(t_2^* - t_1^*) - \gamma_*]^2 + (-2t_2^* + \alpha s_*)^2\} \tag{2.9}$$

$$m_1^* = -(\alpha^2 s_* + \gamma_*)/2 + {}^1/_2\, s_*^{-1}\{[2(t_1^* - t_2^*) + \alpha s_*]^2 + (2t_2^* - \gamma_*)^2\} \tag{2.10}$$

$$m_1^* = -(\alpha^2 s_* + \gamma_*)/2 + {}^1/_2\,\{\gamma_*^{-1}[2(t_1^* - t_2^*) + \alpha s_*]^2 + s_*^{-1}(2t_2^* - \gamma_*)^2\} \tag{2.11}$$

$$m_1^* = -(\alpha_2 s_* + \gamma_*)/2 + {}^1/_2\,\gamma_*^{-1}\{[2(t_1^* - t_2^*) + \alpha s_*]^2 + (2t_2^* - \gamma_*)^2\} \tag{2.12}$$

The surface (Fig. 2) is symmetrical about the origin of the coordinates. The numbers in Fig. 2 denote the corresponding surfaces given by Eqs. (2.1)-(2.12).

The flow rule for any part of the surface presented in Fig. 2 is written in the form

$$\dot{\varepsilon}_{10} : \dot{\varepsilon}_{20} : \dot{k}_1 = \frac{\partial f}{\partial t_1} : \frac{\partial f}{\partial t_2} : \frac{\partial f}{\partial m_1} \tag{2.13}$$

where $k_1 = -1/3\ \dot{\gamma}_1$ and $f(t_1, t_2, m_1) = 0$ is the equation (in coordinates without the asterisks) of this surface.

In the following, when solving the problems, we use an approximation of the surface thus found, this approximation having been obtained by continuing the parts (2.1)-(2.6) (and the parts symmetric to them) up to their intersection. In the case $t_1 = t_2$ the limit relationships obtained by means of regular states have the form

$$m_1^* = -\frac{\alpha^2 s_* + \gamma_*}{2} + \frac{1}{4\gamma_*}(2t^* + \alpha s_* - \gamma_*)^2, \tag{2.14}$$

$$-\frac{\alpha s_* + \gamma_*}{2} \leqslant t^* \leqslant \frac{\gamma_* - \alpha s_*}{2}$$

$$m_2{}^* = \frac{\alpha^2 s_* + \gamma_*}{2} - \frac{1}{4\gamma_*}(-2t^* + \alpha s_* - \gamma_*)^2, \qquad \frac{\alpha s_* - \gamma_*}{2} \leqslant t^* \leqslant \frac{\alpha s_* + \gamma_*}{2} \tag{2.15}$$

$$m_1{}^* = -\frac{\alpha^2 s_* + \gamma_*}{2} + \frac{1}{4s_*}(2t^* + \alpha s_* - \gamma_*)^2, \qquad \frac{\gamma_* - \alpha s_*}{2} \leqslant t^* \leqslant \frac{\alpha s_* + \gamma_*}{2} \tag{2.16}$$

$$m_2{}^* = \frac{\alpha^2 s_* + \gamma_*}{2} - \frac{1}{4s_*}(-2t^* + \alpha s_* + \gamma_*)^2, \qquad -\frac{\alpha s_* + \gamma_*}{2} \leqslant t^* \leqslant \frac{\alpha s_* - \gamma_*}{2} \tag{2.17}$$

$$\begin{gathered} m_2{}^* - m_1{}^* = (\alpha^2 s_* + \gamma_*)/2 - (\alpha s_* - \gamma_*)^2/4\delta, \\ \delta = \gamma_*; \quad \alpha s_* \leqslant \gamma_*, \quad \delta = s_*; \quad \alpha s_* \geqslant \gamma_* \end{gathered} \tag{2.18}$$

The limit relationships obtained by means of singular states (for $\alpha s_* \leq \gamma_*$) have the form

$$\pm m_1{}^* = \frac{1}{2}\left\{\alpha^2 s_* + \gamma_* - \gamma_*\left[\frac{1}{2}\left(\frac{m_2{}^*}{t^*} \mp \frac{2t^*}{\gamma_*}\right)^2 + \frac{\pm 2t^* - \alpha s_* + \gamma_*}{\gamma_*}\left(\frac{m_2{}^*}{t^*} \mp \frac{2t^*}{\gamma_*}\right) + \frac{(\pm 2t^* - \alpha s_* + \gamma_*)^2}{\gamma_*{}^2}\right]\right\}$$

$$\pm m_2{}^* = {}^1/_8\,[\gamma_*^{-1}(\alpha s_* - \gamma_* - \delta_{12}\mp)^2 - s_*^{-1}(\mp 4t^* + \alpha s_* - \gamma_* - \delta_{12}\mp)^2]$$

$$\pm m_2{}^* = -{}^1/_8\,[\gamma_*^{-1}(\mp 2t^* + \alpha s_* - \gamma_* - \delta_1\mp)^2 - s_*^{-1}(\pm 2t^* + \alpha s_* - \gamma_* - \delta_1\mp)^2]$$

$$\pm m_2{}^* = {}^1/_8\,[\gamma_*^{-1}(\pm 2t^* + \alpha s_* - \gamma_* + \beta_1\mp)^2 - s_*^{-1}(\mp 2t^* + \alpha s_* - \gamma_* + \beta_1\mp)^2]$$

The surface is symmetric about the origin of the coordinates.

Analogously to the previous case, in the solution of the problem it is more convenient to use an approximation of the given surface, this approximation having been obtained by continuing its parts (2.14)-(2.18) and those symmetric to them as far as their intersection. In Fig. 3 we have presented such an approximated surface.

Proceeding from the expression for the rate of dissipation of mechanical energy during a plastic flow in the case $t_1 = t_2 = t$, we find that the flow rule for any part of the surface thus obtained will be

$$(\dot{\varepsilon}_{10} + \dot{\varepsilon}_{20}) : \dot{k}_1 : \dot{k}_2 = \frac{\partial f}{\partial t} : \frac{\partial f}{\partial m_1} : \frac{\partial f}{\partial m_2} \tag{2.19}$$

where $\dot{k}_i = -1/2\dot{\gamma}_i$ ($i = 1, 2$) and $f(t, m_1, m_2) = 0$ is the equation of this part.

3. As an example we consider the problem of determining the limit load for a closed cylindrical shell with plane lids at the ends, subjected to an internal pressure of intensity q. We assume that the shell and the circular plate (the lid) are made of a bimetal with identical constituents (i.e., with identical base materials and identical materials of the plating layers), but with different thicknesses of the layers in the cylindrical part and in the lids.

We properly refer the shell to a cylindrical coordinate system with the origin on the surface separating the layers of the bimetal and dividing the generator of the cylinder of length 2L into equal halves. The X axis is directed along the generator; the Z axis is directed along the inner normal of the shell. The plate is referred to an r, Z' coordinate system with the origin on its surface separating the surfaces at the center of the plate. The r axis is assumed to be directed along the radius of the plate; the Z' axis is directed along the normal to the plane separting the layers and directed into the shell.

We introduce the following notation

$$\begin{gathered} x = \frac{X}{L}, \quad z = \frac{2Z}{h_1 + h_2}, \quad \xi = \frac{r}{R}, \quad \zeta = \frac{2Z'}{H_1 + H_2}, \quad \alpha_1 = \frac{h_2}{h_1} \\ \alpha_2 = \frac{H_2}{H_1}, \quad \dot{u}_0 = \frac{\dot{U}_0}{L}, \quad \dot{w}_0 = \frac{2\dot{W}_0}{h_1 + h_2}, \quad \dot{u}_n = \frac{\dot{U}_n}{R}, \\ \dot{w}_n = \frac{2W_n}{H_1 + H_2}, \quad p = \frac{qR}{\sigma_0 h_1 (1 + \alpha_1)} \end{gathered} \tag{3.1}$$

where h and h_{12} are the height of the upper and lower layers of the shell; H_1, H_2 are the corresponding quantities for the plate (the layer located in the positive direction of the ζ axis is taken as the lower layer); R is the radius of the plate (the shell); $\dot{U}_0$, $\dot{W}_0$, $\dot{U}_n$, $\dot{W}_n$ are the rates of displacement of points on the surface separating the layers, in the direction of the x, z, ξ, ζ axes for the shell and the plate respectively.

Taking into account the notation (3.1), we write relationships of the Kirchhoff–Love hypothesis in the form:

a) for the shell

$$\dot{\varepsilon}_1 = \dot{\varepsilon}_{10} - z\dot{\gamma}_1, \quad \dot{\varepsilon}_{10} = \frac{d\dot{u}_0}{dx}, \quad \dot{\gamma}_1 = \left(\frac{h_1}{L}\right)^2 \frac{(1+\alpha_1)^2}{4} \frac{d^2\dot{w}_0}{dx^2},$$
$$\dot{\varepsilon}_2 = \dot{\varepsilon}_{20} = -\frac{1+\alpha_1}{2}\frac{h_1}{R}\dot{w}_0 \tag{3.2}$$

b) for the plate

$$\dot{\varepsilon}_r = \dot{\varepsilon}_{r0} - \zeta\dot{\gamma}_r, \quad \dot{\varepsilon}_\theta = \dot{\varepsilon}_{\theta 0} - \zeta\dot{\gamma}_\theta, \quad \dot{\varepsilon}_{r0} = \frac{d\dot{u}_n}{d\xi}, \quad \dot{\varepsilon}_{\theta 0} = \frac{\dot{u}_n}{\xi}$$
$$\dot{\gamma}_r = \left(\frac{H_1}{R}\right)^2 \frac{(1+\alpha_2)^2}{4}\frac{d^2\dot{w}_n}{d\xi^2}, \quad \gamma_\theta = \left(\frac{H_1}{R}\right)^2 \frac{(1+\alpha_2)^2}{4}\frac{1}{\xi}\frac{d\dot{w}_n}{d\xi} \tag{3.3}$$

We consider three possible cases where the load-carrying capacity is fully taken up by the construction:

1) the shell is in the limit state (the plate is rigid);

2) the plate is in the limit state (the shell is rigid);

3) the entire construction is in the limit state.

To find the relationships between the parameters of the shell and the plate which lead to the cases enumerated above, we must solve each problem separately.

1. A Cylindrical Shell with Rigidly Built-in Contour That Can Move in the Direction of the Generator. The shell is loaded by an internal pressure of intensity q and a force $T_0 = qR/2$ uniformly distributed over the ends and pulling along the generator.

The equations of equilibrium of an element of the shell are

$$\frac{dt_1}{dx} = 0, \quad \frac{d^2m_1}{dx^2} + 2\frac{\beta_2\beta_3^2}{\beta_1}(t_2 - p) = 0$$
$$\beta_1 = \frac{h_1(1+\alpha_1)}{H_1(1+\alpha_2)}, \quad \beta_2 = \frac{'2R}{H_1(1+\alpha_2)}, \quad \beta_3 = \frac{L}{R} \tag{3.4}$$

In view of symmetry of the problem, in the following we consider only one half of the shell $0 \le x \le 1$.

The boundary conditions are written in the form

$$Q = dm_1 / dx = 0, \; \dot{u}_0 = 0, \; d\dot{w}_0 / dx = 0, \; x = 0 \tag{3.5}$$

$$t_1 = t_0 = \frac{T_0}{\sigma_0 h_1(1+\alpha_1)} = \frac{p}{2}, \quad \dot{w}_0 = 0, \quad \frac{d\dot{w}_0}{dx} = 0, \quad x = 1$$
$$Q = 4LQ^\circ / \sigma_0 h_1^2 (1+\alpha_1)^2 \tag{3.6}$$

where Q° is the shear force.

From the loading conditions of the shell we draw the conclusion that, in its limit state, a plastic state approximated by a yield surface that is analogous to the part (2.5) (see Fig. 2) is fulfilled:

$$t_2 = t_{20} = [\alpha_1(s_* - \Delta_2) + \gamma_* - \Delta_1] / (1 + \alpha_1) \tag{3.7}$$

Using the expressions (2.13), (3.5)-(3.7), we find the flow rule

$$\dot{\varepsilon}_{10} = 0, \; \dot{\gamma}_1 = 0, \; \dot{u}_0 = 0, \; \dot{w}_0 = A(x - 1)$$

Since there is no possibility of satisfying the last one of the boundary conditions (3.5), (3.6), the circles x=0 and x=1 will be hinged [8]. Thus, instead of these boundary conditions we must satisfy the following conditions [see (2.1)-(2.4)]:

$$m_1 = \frac{4}{(1+\alpha_1)^2}\left\{-\frac{\alpha_1^2(s_* + \Delta_2) + \gamma_* - \Delta_1}{2} + \frac{1}{4s_*}[(1+\alpha_1)t_1 + \Delta_1 + \alpha_1(s_* + \Delta_2) - \gamma_*]^2\right\} \tag{3.8}$$

if $\gamma_* - \alpha_1 s_* \leqslant (1+\alpha_1)\, t_1 + \Delta_1 + \alpha_1\Delta_2 \leqslant \gamma_* + \alpha_1 s_*$

$$m_1 = \frac{4}{(1+\alpha_1)^2}\left\{-\frac{\alpha_1^2(s_*+\Delta_2)+\gamma_*-\Delta_1}{2} + \frac{1}{4\gamma_*}[(1+\alpha_1)t_1+\Delta_1+\alpha_1(s_*+\Delta_2)-\gamma_*]^2\right\}$$

if $0 \leqslant (1+\alpha_1)\, t_1 + \Delta_1 + \alpha_1\Delta_2 \leqslant \gamma_* - \alpha_1 s_*$ for $x = 0$

$$m_1 = \frac{4}{(1+\alpha_1)^2}\left\{\frac{\alpha_1^2(s_*-\Delta_2)+\gamma_*+\Delta_1}{2} - \frac{1}{4\gamma_*}[-(1+\alpha_1)t_1-\Delta_1+\alpha_1(s_*-\Delta_2)-\gamma_*]^2\right\} \text{ for } x = 1 \tag{3.9}$$

Solving the equations of equilibrium (3.4) with the conditions (3.7) and satisfying the boundary conditions at the center of the shell and the first condition (3.6), we find

$$m_1 = \frac{\beta_2\beta_3^2}{\beta_1}x^2(p-t_{20}) + \frac{4}{(1+\alpha_1)^2}\left[-\frac{\alpha_1^2(s_*+\Delta_2)+\gamma_*-\Delta_1}{2} + \frac{1}{4s_*}(p^*+\alpha_1 s_*-\gamma_*)^2\right], \quad t_1 = \frac{p}{2} \tag{3.10}$$

Here it was assumed that

$$p^* = {}^1/_2\,(1+\alpha_1)\,p + \Delta_1 + \alpha_1\Delta_2 \geqslant \gamma_* - \alpha_1 s_*$$

i.e., the condition (3.8) is satisfied.

Substituting the expression for m_1 from (3.10) into Eq. (3.9), for the finding of the limit load parameter we obtain the following quadratic equation

$$p^{*2}\frac{s_*+\gamma_*}{4s_*\gamma_*} + p^*\left[(1+\alpha_1)\frac{\beta_2\beta_3^2}{2\beta_1} + (\alpha_1 s_*-\gamma_*)\frac{\gamma_*-s_*}{2s_*\gamma_*}\right] - (\alpha_1^2 s_*+\gamma_*) -$$
$$-(1+\alpha_1)\frac{\beta_2\beta_3^2}{4\beta_1}[\alpha_1(s_*+\Delta_2)+\gamma_*+\Delta_1] + (\alpha_1 s_*-\gamma_*)^2\frac{s_*+\gamma_*}{4s_*\gamma_*} = 0$$

When $p^* = \gamma^* + \alpha_1 s_*$, the shell fails due to the action of the axial load.

2. A Circular Plate Rigidly Fixed along the Outline, Subjected to an Internal Pressure of Intensity q (the case where $\alpha_2 s_* \leq \gamma_*$).

The equations of equilibrium of an element of the plate are

$$\frac{d}{d\xi}(\xi t_r) - t_\theta = 0, \quad \frac{d}{d\xi}(\xi m_r) - m_\theta = Q_*\xi,$$
$$Q_* = \frac{4Q_0R}{\sigma_0(H_1+H_2)^2} = \beta_1\beta_2 p\xi \tag{3.11}$$

where Q_0 is the shear force. We assume that in the limit state of the plate the stress state corresponds to the condition $t_r = t_0$. Then from the first equation of equilibrium from (3.11) we find $t_r = t_\theta = t = \text{const} > 0$ (0 $(0 \leq \xi \leq 1)$.

The boundary conditions of the problem are

$$t_r = t_\theta, \quad m_r = m_\theta, \quad Q_0 = 0 \text{ for } \xi = 0 \tag{3.12}$$
$$\dot{u}_n = 0, \quad \dot{w}_n = 0, \quad d\dot{w}_n/d\xi = 0 \text{ for } \xi = 1 \tag{3.13}$$

From the loading conditions we conclude that in the limit state of the plate the following plastic states are realized [see Fig. 3 and (2.14)-(2.18)]

for $0 \leqslant \tau \leqslant \gamma_* - \alpha_2 s_*$, $\tau = (1+\alpha_2)\,t + \Delta_1 + \alpha_2\Delta_2$

$$0 \leqslant \xi \leqslant \rho : m_\theta = \frac{4}{(1+\alpha_2)^2}\left[-\frac{\alpha_2^2(s_*+\Delta_2)+\gamma_*-\Delta_1}{2} + \frac{1}{4\gamma_*}(\tau+\alpha_2 s_*-\gamma_*)^2\right] \tag{3.14}$$

$$\rho \leqslant \xi \leqslant 1 : m_\theta - m_r = \frac{2}{(1+\alpha_2)^2}\left[\frac{1}{2\gamma_*}(\alpha_2 s_*-\gamma_*)^2 - \alpha_2^2 s_* - \gamma_*\right] \tag{3.15}$$

for $\gamma_* - \alpha_2 s_* \leqslant \tau \leqslant 2\delta$

$$0 \leqslant \xi \leqslant \rho : m_\theta = \frac{2}{(1+\alpha_2)^2}\left[\frac{1}{2s_*}(\tau + \alpha_2 s_* - \gamma_*)^2 - \alpha_2{}^2(s_* + \Delta_2) - \gamma_* + \Delta_1\right] \tag{3.16}$$

$\rho \le \xi \le 1$: the condition (3.15)

for $2\delta \le \tau \le \alpha_2 s_* + \gamma_*$

$0 \le \xi \le 1$: the condition (3.16).

The coordinate $t^* = \delta$ of the point H (see Fig. 3) is determined according to the expression

$$\delta = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad a = \frac{s_* + \gamma_*}{s_* \gamma_*}, \quad b = \frac{(\gamma_* - s_*)(\alpha_2 s_* - \gamma_*)}{s_* \gamma_*},$$
$$c = \frac{(\alpha_2 s_* - \gamma_*)^2}{4s_*} - \frac{\alpha_2{}^2 s_* + \gamma_*}{2}$$

In the following only Case 1 is considered. For this, using the relationships (2.19), (3.3), (3.14), (3.15) and satisfying the continuity conditions for the displacements and $dw_n/d\xi$ for $\xi = \rho$, as well as the first two conditions from (3.13), we find

$$\begin{aligned} &t = (1+\alpha_2)^{-1}[\gamma_* - \Delta_1 - \alpha_2(s_* + \Delta_2)] \\ &\dot{u}_n = 0, \quad \dot{w}_n = \dot{w}_*\,[\xi\rho^{-1}(\rho \ln \rho - 1) + 1] \ \text{ for } 0 \leqslant \xi \leqslant \rho \\ &\dot{u}_n = 0, \quad \dot{w}_n = \dot{w}_* \rho \ln \xi \ \text{ for } \rho \leqslant \xi \leqslant 1 \end{aligned} \tag{3.17}$$

From the relationship (3.17) we see that Case 1 is considered with justification, since the required inequality for it is fulfilled.

Integrating the equations of equilibrium in each of the zones and satisfying the continuity conditions for the transition of the quantity m_r, through $\xi = \rho$, and also taking into account the relationship (3.14), after having satisfied the boundary conditions (3.12), we obtain

$$\begin{aligned} m_r &= \frac{2}{(1+\alpha_2)^2}\left[\Delta_1 - \gamma_* - \alpha_2{}^2(s_* + \Delta_2) + \frac{1}{2\gamma_*}(\tau + \alpha_2 s_* - \gamma_*)^2\right] + \frac{1}{3}\beta_1\beta_2 p\xi^2 \ \text{ for } 0 \leqslant \xi \leqslant \rho \\ m_r &= \frac{2}{(1+\alpha_2)^2}\left\{\left[\frac{1}{2\gamma_*}(\alpha_2 s_* - \gamma_*)^2 - \alpha_2{}^2 s_* - \gamma_*\right]\left(\frac{3}{2} + \ln\frac{\xi}{\rho}\right) + \right. \\ &\left. + \frac{\tau}{\gamma_*}\left(\frac{\tau}{2} + \alpha_2 s_* - \gamma_*\right) - \alpha_2{}^2\Delta_2 + \Delta_1\right\} + {}^1\!/_2\beta_1\beta_2 p\xi^2 \ \text{ for } \rho \leqslant \xi \leqslant 1 \\ \rho &= \left\{\frac{6}{p\beta_1\beta_2(1+\alpha_2)^2}\left[\alpha_2{}^2 s_* + \gamma_* - \frac{(\alpha_2 s_* - \gamma_*)^2}{2\gamma_*}\right]\right\}^{1/2} \end{aligned} \tag{3.18}$$

Since it is not possible to satisfy the condition $(d\dot{w}_n/d\xi)$ $(\xi = 1) = 0$, for $\xi = 1$ there exists a plastic hinge [8] and the boundary condition has the form

$$m_r = \frac{4}{(1+\alpha_2)^2}\left[\frac{\alpha_2{}^2(s_* - \Delta_2) + \gamma_* + \Delta_1}{2} - \frac{(\alpha_2 s_* - \gamma_*)^2}{\gamma_*}\right] \ \text{ for } \xi = 1$$

From the last condition we find that

$$p = \frac{4}{(1+\alpha_2)^2\beta_1\beta_2}\left[(\alpha_2{}^2 s_* + \gamma_*)(2.5 - \ln\rho) - \frac{(\alpha_2 s_* - \gamma_*)^2}{2\gamma_*}(4.5 - \ln\rho)\right]$$

3. A Cylindrical Shell with Plane Lids. In this case the expressions for the moments in each of the consituent constructions are the same as in the first two cases. To find the limit load parameters we must satisfy the continuity condition of the moments (M_1, M_r) on the joint of the constructions, and also the condition

$$\frac{h_1{}^2}{4L}(1+\alpha_1)^2\frac{dm_1}{dx}(x=1) = tH_1(1+\alpha_2)$$

(the condition that the shear force in the shell $(x=1)$ and the tensile force in the plate $(\xi = 1)$ are equal).

Finally, to find limit load parameter in Cases 1-3 (see 3.14)-(3.16) we find:

Case 1 for $\nu = 0$ and Case 2 for $\nu = 1$

$$\begin{aligned} \Phi &= \frac{2}{(1+\alpha_2)^2}\left\{\left[\frac{1}{2\gamma_*}(\alpha_2 s_* - \gamma_*)^2 - \alpha_2{}^2 s_* - \gamma_*\right]\left(\frac{3}{2} - \ln\rho\right) + \right. \\ &\left. + \frac{\tau}{\gamma_*}\left(\frac{\tau}{2} + \alpha_2 s_* - \gamma_*\right) - \alpha_2{}^2\Delta_2 + \Delta_1 - \nu(\alpha_2 s_* - \gamma_*)^2\frac{s_* - \gamma_*}{2\gamma_* s_*}\right\} + \frac{1}{2}\beta_1\beta_2 p \end{aligned}$$

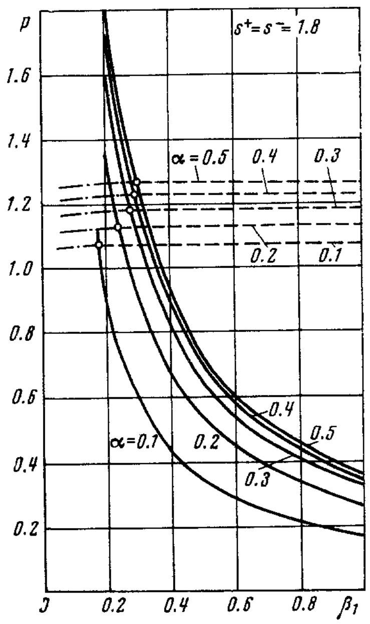

Fig. 4

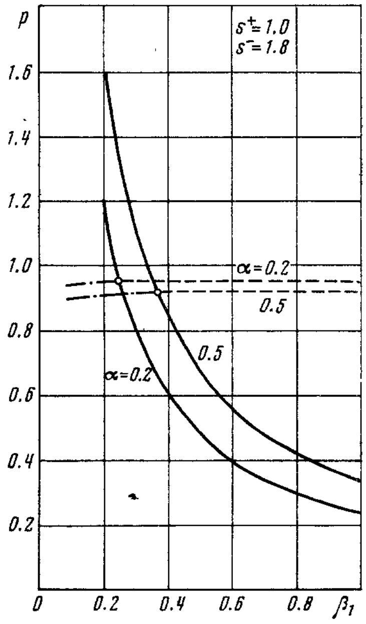

Fig. 5

and Case 3

$$\Phi = \frac{2}{(1+\alpha_2)^2}\left[\frac{1}{2\gamma_*}(\tau + \alpha_2 s_* - \gamma_*)^2 - \alpha_2^2(s_* + \Delta_2) + \Delta_1 - \gamma_*\right] + \frac{1}{3}\beta_1\beta_2 p$$

where ρ is determined from the expression (3.18), while

$$\Phi = \beta_1\left\{\beta_2\beta_3^2\left[p - \frac{\alpha_1(s_* - \Delta_2) + \gamma_* - \Delta_1}{1+\alpha_1}\right] + \frac{4\beta_1}{(1+\alpha_1)^2}\left[\frac{1}{4s_*}\left(\frac{1+\alpha_1}{2}p + \Delta_1 + \alpha_1\Delta_2 + \alpha_1 s_* - \gamma_*\right)^2 - \frac{\alpha_1^2(s_* + \Delta_2) + \gamma_* - \Delta_1}{2}\right]\right\}$$

$$t = \beta_1\beta_3\left[p \mid \frac{\alpha_1(s_* - \Delta_2) + \gamma_* - \Delta_1}{1+\alpha_1}\right]$$

In Figs. 4 and 5 we have represented the graphs of the limit load depending on the geometry parameters β_1 and $\alpha = \alpha_1 = \alpha_2$ in the case $\beta_2 = 20$, $\beta_3 = 5.0$, $\gamma^+ = \gamma^- = 1$. Solid lines denotes the graphs for the limit load of the rigidly fixed plate, the dashed lines denote them for the limit load of the cylindrical shell with rigid lids, and the dash-dotted lines denote them for the case when the plate and the shell are in the limit state.

If we consider the limit state of the construction as a whole, then for the given geometrical parameters, in the role of the limit load we must take the least of the three values of p. For those values of the parameter for which the limit load for the rigidly fixed plate and the cylindrical shell with rigid ends coincide, the entire construction is in the limit state. The construction parameters for which the last case of limit state is realized should be considered optimum. On the graphs the corresponding points are denoted by circles.

If the dash-dotted line runs below the rest, then in these cases the entire construction is also in the limit state.

When $\alpha = \alpha_1 = \alpha_2$ increases (an increase of the thickness of the layer of the stronger material, maintaining the thickness of the construction as a whole), the value of the limit load increases.

From the comparison of the graphs of Figs. 4 and 5 we see that a replacement of the isotropic material of one of the layers with a material having different moduli, with a reduced yield point in tension, leads to a considerable reduction in the limit load of the optimal design. But such reduction percentagewise is below the reduction of the yield point. Thus, a reduction of the yield point of the lower layer by 44% leads to a reduction of the limit load by 27%, when $\alpha = 0.5$; it amount to 10%, when $\alpha = 0.1$.

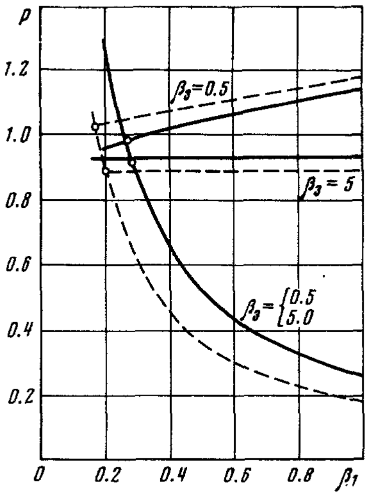

Fig. 6

In Fig. 6 we have represented the results of calculations to find the optimal parameters for a two-layered construction made of stainless steel (d=7.7 g/cm^3, $\sigma_s^+=\sigma_s^-=70$ kg/mm^2) and an aluminum alloy (d=2.85 g/cm^3, $\sigma_s^+=\sigma_s^-=59.5$ kg/mm^2). We have taken $\alpha=\alpha_1=\alpha_2=0.85$, $\beta_2=20$. The solid lines are plotted for the case where the upper layer is made of the stainless steel, while the lower layer is made of the aluminum alloy. The dashed lines denote the curves for a single-layered construction of stainless steel having the same weight as the two-layered construction. If the arrangement of the layers of the bimetallic construction is altered so that the upper layer will be of the aluminum alloy, while the lower will be of stainless steel, then in comparison with the preceding case, for small values of β_1, the limit load is almost unaltered, while for $\beta_1 > 0.4$ it is higher by approximately 4-5%. The corresponding curves practically merge with those presented in Fig. 6.

Comparing the values of the limit loads for the optimal designs, we can conclude that the replacement of the single-layered construction with a two-layered construction of the same weight, when one of the layers has a reduced yield point, does not lead to a significant reduction of the limit load, and in certain cases it can lead to an increase in it. Thus, in the case considered a reduction in the yield point by 15% leads to a reduction of 4% in the limit load of the optimal design for $\beta_3=0.5$. When $\beta_3=5$, it is 3.3% higher than the limit load for a single-layered steel construction.

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